Reassessing the Ubiquity of Small-World Networks

Qawi K. Telesford[1], Karen E. Joyce[1], Satoru Hayasaka[2,3], Jonathan H. Burdette[3], Paul J. Laurienti[3]

[1] School of Biomedical Engineering and Sciences, Virginia Tech-Wake Forest University, Winston-Salem, NC
[2] Department of Biostatistical Sciences, Wake Forest University School of Medicine, Winston-Salem, NC
[3] Department of Radiology, Wake Forest University School of Medicine, Winston-Salem, NC

Virginia Tech Wake Forest University
School of Biomedical Engineering and Sciences

1. Introduction

Network science is an emerging field that is gaining popularity in fields like neuroimaging. Using a graph metric approach to study the brain provides a global approach to investigate functional connectivity. In building a brain network, it is important to study its processing properties. Networks with high interconnectivity and high efficiency are considered "small-world" networks.

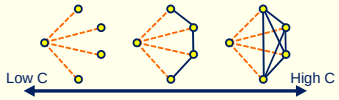
Although the concept of the small-world networks has been around for years[1], a quantitative measure of small-worldness has only emerged recently[2]. The small-world coefficient, σ, has gained considerable popularity as a quantitative measure of small-worldness, but it can result in aberrant findings. We propose a new small-world metric, ω, which addresses some of these problems and provides a more accurate measure of network small-worldness.

2. What Is a Small-World Network?

"Highly clustered, like a regular lattice, yet has small characteristic path length, like random graph."[1]

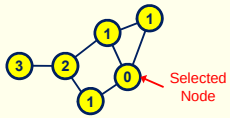
Clustering Coefficient (C)

A measure of local neighborhood connectivity, calculated as the likelihood that each of the neighbors of a node is also a neighbor.

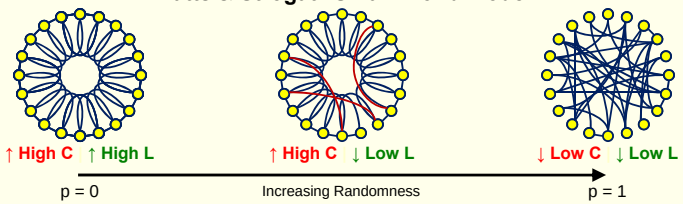


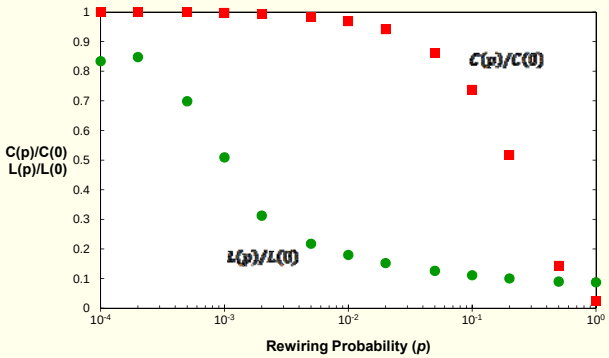
Path Length (L)

A measure of the distance between nodes on the network, calculated as the shortest distance between each node and every other network node.



Watts & Strogatz Small-World Model





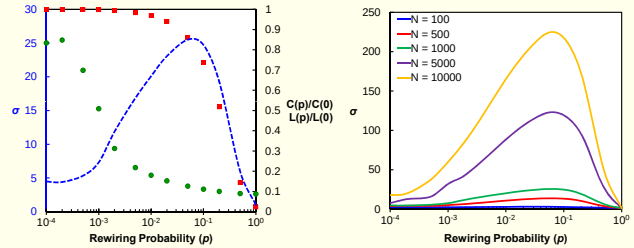
3. Quantifying Network Small-Worldness in Simulated Networks

Compared to σ, the small-world metric, ω, more accurately follows the Watts & Strogatz model.

Small-World Coefficient (σ)

- Clustering compared to equivalent ***random network***
- Path length compared to equivalent ***random network***
- $\sigma > 1$ considered small-world
- As network size grows, values of σ grow

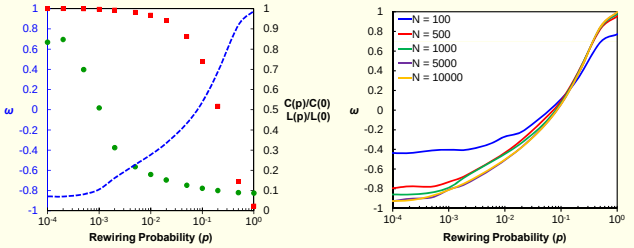
$$\sigma = \frac{C/C_{rand}}{L/L_{rand}}$$



Small-World Metric (ω)

- Clustering compared to equivalent **lattice network**
- Path length compared to equivalent **random network**
- Inherently scaled from -1 to 1
- ω values closer to 0 are considered small-world
- $\omega < 0$ are more like a lattice network
- $\omega > 0$ are more like a random network

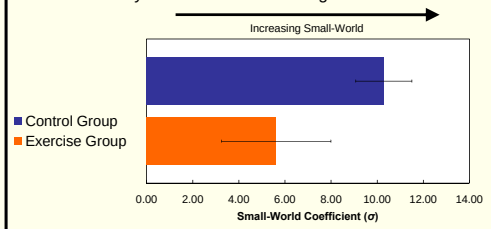
$$\omega = \frac{L_{rand}}{L} - \frac{C}{C_{latt}}$$

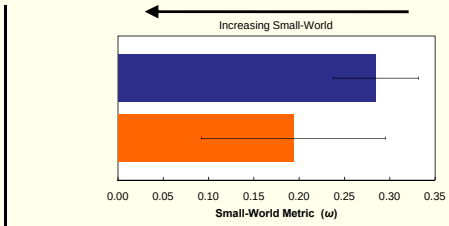


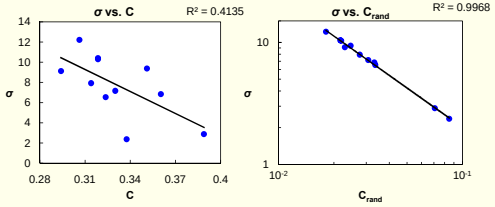
4. Small-World Analysis in Real Neuroimaging Data

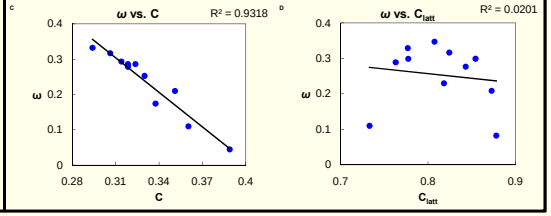
Study comparing network changes due to an exercise regime found slight, increase but no significant differences both for clustering and path length between control group (n=5) and exercise group (n=6)

- σ shows significant difference between groups, suggests that control group is more small-world
- ω shows no significant difference between groups, suggests that exercise group is more small-world
- ω more readily characterizes clustering in the network while σ shows undue influence to clustering in equivalent random network (C_{rand})









References

1. Watts DJ and Strogatz SH, *Collective Dynamics of 'Small-World' Networks*. **Nature**, 1998. 393(6684): p. 440-442.
2. Humphries MD, Gurney K, and Prescott TJ, *The Brainstem Reticular Formation Is a Small-World, Not Scale-Free, Network*. **Proceedings of the Royal Society B: Biological Sciences**, 2006. 273(1585): p. 503-511.

Supported by:

Wake Forest University (GCRC (M01-RR07122)), National Institutes on Aging (AG029285-01A1) National Institute of Neurological Disorders and Stroke (NS070917-01), National Institute on Alcohol Abuse and Alcoholism (AA017710-01)